# Comparison of the  $C^{13}(d,\alpha)B^{11}$  and  $B^{11}(He^3, p)C^{13}$  Reactions

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Angular distributions have been measured for  $C^{18}(d, \alpha)B^{11}$  (ground state) at 3.35, 3.55, 3.85, and 4.20 MeV. Differential excitation functions for this reaction were measured in 100-keV intervals over the 3.1- to 4.2-MeV region at 54°, 84°, 119°, and 154°. Angular distributions for the  $B^{11}(He^3, p)C^{13}$  (ground state) reaction have been measured at 8.6, 9.6, and 10.3 MeV. Differential excitation functions were measured in 200-keV intervals over the 8.0- to 11.0-MeV region at 35° and 140°. The two reactions studied are related in that both involve the transfer of a deuteron between  $B<sup>11</sup>$  and  $C<sup>13</sup>$  in a common momentum transfer region. Notwithstanding, the two reactions appear to proceed predominantly by different mechanisms, viz. compound-nucleus formation and direct process, respectively. The  $C^{13}(d, \alpha)B^{11}$  data are fitted using Legendre polynomials in cose. The forward portions of angular distributions of  $B^{11}(He^{3},\phi)C^{13}$  are fitted using two versions of the plane-wave Born approximation theory.

### **INTRODUCTION**

ANGULAR distributions and differential excitation functions have been measured for two reactions,  $C^{13}(d, \alpha)B^{11}$  and  $B^{11}(He^3, p)C^{13}$ , in an attempt to obtain information on the mechanism involved in two-particle transfer reactions. The reactions are related in that both involve the transfer of a deuteron between  $B<sup>11</sup>$  and  $C<sup>13</sup>$ .

The experiments were performed using the beam from the Rochester 27-in. cyclotron, which provided deuterons of 3 to 4 MeV and He<sup>3</sup> ions of 8 to 11 MeV. These ranges of energy provided an overlap in the momentum transfer for the two reactions.

While most of the experimental apparatus was conventional, one device, which was developed in the course of the present work, was novel. This is an array of solidstate detectors which replaces nuclear emulsions in the image plane of a broadrange spectrograph magnet. The chronotron principle is used in conjunction with the array, so as to dispense with the need for separate amplifiers for each detector. Outputs from individual detectors are fed into successive points of a lumped delay line. The position of a counter which detects a given event is determined from the difference in arrival times of the pulse at the two ends of the delay line. This system is described elswhere.<sup>1</sup>

For the  $B^{11}(He^3, p)C^{13}$  reaction, the angular distributions of the emitted protons show pronounced forward peaking, and the excitation functions show only slow variation with energy. This suggests that the reaction proceeds by a direct process, and the data have been analyzed on this basis, using two different plane-wave Born approximation (PWBA) approaches. Qualitative fits to the angular distributions have been obtained using the customary Bessel function dependence. In the other approach, numerically integrated curves were obtained following an alternate PWBA theory developed for two-particle stripping reactions by French and Macfarlane. These latter results were found to bear a striking resemblance to the former more conventional curves.

The angular distributions of the emitted alpha particles in the  $C^{13}(d, \alpha)B^{11}$  reaction show pronounced symmetry about 90°. The excitation functions show some structure, but no sharp resonances appear. These results indicate that the reaction may proceed through several broad levels in the compound nucleus. To examine this possibility, least-squares fits to the angular distributions were made, using Legendre polynomials. Good fits were obtained by including terms through seventh order. On the basis of nuclear penetration factors, such high orders are possible, but they imply a strong excitation of the relevant levels in the compound nucleus.

An important difference in the angular distributions for the two cases was expected on the basis of spin selection rules, which permit both the triplet and singlet deuteron transfer in the case of (He<sup>3</sup> , *p)* and only triplet in the case of the  $(d, \alpha)$  reaction. However, this effect could not be analyzed since the  $(d, \alpha)$  reaction did not appear to proceed as a direct process. This shows that the overlap of momentum transfer regions of two related reactions does not necessarily imply similar reaction mechanisms, at least at energies used in this work.

### **EXPERIMENTAL PROCEDURE**

#### **Beam**

The beam used for these experiments was provided by the Rochester 27-in. variable-energy cyclotron.<sup>2</sup> The layout of the cyclotron and associated equipment is shown in Fig. 1. The beam, after being electrostatically deflected from the magnetic field of the cyclotron, passes through a set of steering magnets and a set of quadrupole magnets which focus it onto a defining slit. This slit provides the object for a double-focusing analyzer magnet, which focuses the image on a target located at the center of a scattering chamber. The analyzer field is measured with precision by means of a nuclear magnetic resonance unit. The energy of the beam can

<sup>&</sup>lt;sup>1</sup>O. M. Bilaniuk, B. B. Marsh, A. K. Hamann, and J. C. Heurtley, Nucl. Instr. Methods 14, 63 (1961).

<sup>2</sup> H. W. Fulbright, D. A. Bromley, J. A. Bruner, A. K. Hamann, and R. A. Hawrylak, University of Rochester Report, NYO-6541 1954 (unpublished).



FIG. 1. Layout of the Rochester 27-in. cyclotron and associated equipment.

then be determined using calibration points obtained from sharp resonances in various nuclear reactions. For the experiments described here the cyclotron provided He<sup>3</sup> beams over an energy range of 8 to 11 MeV with an intensity of about  $0.03 \mu A$ , and deuteron beams over an energy range of 3.0 to 4.2 MeV with an intensity of about 0.2  $\mu$ A. In both cases the beam area was 3 mm X6 mm and the beam energy spread was approximated 30 keV.

For the purpose of determining absolute cross sections, the integrated beam current was measured either by determining the charge collected at the target, as in the case of targets with thick backings, or at the integrator cup positioned behind the target, as in the case of thin self-supporting targets. The charge collected was drained to ground through a precision resistor and the resultant voltage, proportional to the beam current, was amplified by a dc amplifier to give a signal large enough to be conveniently and accurately integrated.

### **Targets**

To produce the boron targets, natural boron  $(81.3\%$  B<sup>11</sup>) was evaporated from a tungsten filament and deposited on a 3-mil-thick tantalum backing. The high temperatures and long evaporation times resulted in an appreciable deposition of tungsten along with the boron; however, the amount was not enough to introduce serious experimental difficulties.

The carbon targets were prepared by cracking methyl iodide with a carbon content enriched to  $55\%$  C<sup>13</sup>. The cracking was accomplished by heating a tantalum strip in a methyl iodide atmosphere at a pressure of about 10 cm of mercury. Upon cooling the tantalum strip, the carbon which had deposited would flake off, usually producing one or more target-sized strips. Self-supporting carbon targets of about 0.2 mg/cm<sup>2</sup> were produced by this method. The target thicknesses were determined by measuring the energy loss of 7-MeV alpha particles passing through them. In the case of the self-supporting

carbon target, the energy loss for the two traversals was found to be  $266 \pm 14$  keV. This implies a target thickness of  $0.18 \pm 0.2$  mg/cm<sup>2</sup>.

The results for the boron target were complicated by the presence of the tungsten. A method was devised<sup>3</sup> by which the thickness of the boron content alone could be accurately singled out; it was found to be  $0.30\pm0.04$ mg/cm<sup>2</sup> .

## **Detection**

The angular distributions of protons were measured using a small scattering chamber with a mylar window. The protons were detected by a thin sodium-iodide crystal mounted on a photomultiplier tube which could be swung through an angular range of 0° to 160° relative to the incident beam. A similar detector was placed at  $-140^{\circ}$  as a monitor.

Two different arrangements were used in measuring the alpha-particle angular distributions. Part of the data were taken using a gold-silicon surface-barrier counter mounted inside a small scattering chamber. The detector could be moved under vacuum to permit measurements through an angular range of 5° to 165°. Monitoring was provided by a sodium-iodide crystal positioned at  $-30^{\circ}$  which detected protons emitted from the  $C^{13}(d, p)C^{14}$  reaction. The ground-state alphas emitted from the  $C^{13}(d, \alpha)B^{11}$  reaction were easily resolved at all angles greater than 25°. However, at small angles the detection system became swamped by elastically scattered deuterons, and the alpha peak could not be resolved. To extend the measurements to the forward angles, use was made of the bank of solid-state detectors<sup>1</sup> in the focal plane of the broad-range spectrograph magnet.<sup>4</sup> The magnet was coupled to a scattering chamber which provides for variation of the observation

<sup>3</sup>B. B. Marsh, Ph.D. thesis, University of Rochester, 1962 (unpublished), pp. 8-11.

<sup>4</sup> W. P. Alford, O. M. Bilaniuk, and R. A. Hawrylak, University of Rochester Report, NYO-9683, 1961 (unpublished).

angle over a continuous range without breaking vacuum.<sup>5</sup> This method was used to measure angular distributions over an angular range of 10° to 90° at three energies. Perfect agreement prevailed in the angular range where the two sets of data overlapped.



FIG. 2. Angular distributions of alpha particles emitted in the  $C^{13}(d, \alpha)$ B<sup>11</sup> reaction as recorded at 4.20, 3.95, 3.85, 3.55, and 3.35 MeV. The statistical errors for the data points lie within the circles except where indicated. Solid lines are least-squares Legendre polynomial fits computed on the assumption that the reaction proceeds mainly via compound-nucleus formation.

6 J. W. Verba and R. A. Hawrylak, Rev. Sci. Instr. 32, 1037 (1961).



FIG. 3. Differential yield curves for the  $C^{13}(d, \alpha)B^{11}$  (ground state) reaction as recorded at 100-keV intervals at laboratory angles of 154°, 119°, 84°, and 54° over the energy range from 3.0 to 4.2 MeV.

## **RESULTS AND ANALYSIS**

# $\mathbf{C}^{13}(\boldsymbol{d},\boldsymbol{\alpha})\mathbf{B}^{11}$

Angular distributions of the ground-state alpha particles emitted in the C<sup>13</sup> $(d, \alpha)$ B<sup>11</sup> reaction were measured at five energies: 3.35, 3.55, 3.85, 3.95, and 4.20 MeV. These are shown in Fig. 2. Differential excitation functions were measured at four angles: 54°, 84°, 119°, and 154°. These are shown in Fig. 3.

The near symmetry about 90° in all except the 3.35- MeV data is suggestive of a reaction proceeding predominantly via compound-nucleus formation. The absence of sharp resonances in the excitation function may indicate that the reaction proceeds through the excitation of several broad levels in the compound nucleus. The bombarding energies produced excitations of about  $20$  MeV in the compound nucleus,  $N^{15}$ . Only a limited number of levels is involved in the reaction since the anisotropy of the angular distributions indicates incomplete statistical averaging.

The solid curves in Fig. 2 are least-squares fits to the angular distributions, using Legendre polynomials in  $\cos\theta$ . The fitting was done on an IBM 650 computer by means of a least-squares program obtained from Lawrence Radiation Laboratory.<sup>6</sup> The coefficients obtained

K. Goodwin, Lawrence Radiation Laboratory Report, UCRL-9263, 1960 (unpublished).

$E_d$ (MeV)	3.35	3.55	3.85	3.95	4.20
$(\chi^2/d)^{1/2}$	1.49	1.31	0.94	1.28	1.39
a <sub>0</sub>	$5.76 + 0.03$	$7.45 + 0.05$	$7.82 + 0.04$	$6.85 + 0.05$	$5.90 + 0.05$
a <sub>1</sub>	$1.32 + 0.07$	$-0.57 + 0.12$	$0.48 + 0.07$	$1.60 + 0.14$	$2.15 + 0.12$
a <sub>2</sub>	$4.23 + 0.10$	$2.53 + 0.19$	$3.70 + 0.10$	$0.90 + 0.28$	$3.21 + 0.18$
$a_{3}$	$4.61 + 0.12$	$2.16 + 0.23$	$0.47 + 0.12$	$0.28 + 0.24$	$1.07 + 0.23$
a <sub>4</sub>	$5.25 + 0.27$	$5.34 + 0.27$	$4.17 + 0.16$	$2.20 + 0.42$	$4.66 + 0.25$
a <sub>5</sub>	$2.15 + 0.15$	$0.00 + 0.26$	$-1.90 + 0.19$	$-1.53 + 0.29$	$-1.64 + 0.25$
a <sub>6</sub>	$-0.86 + 0.14$	$-2.20 + 0.16$	$-2.15 + 0.16$	$-2.98 + 0.39$	$-1.46 + 0.24$
a <sub>7</sub>	$-0.10 + 0.15$	$0.36 + 0.23$	$1.50 + 0.17$	$1.68 + 0.25$	$0.87 + 0.22$

TABLE I. Coefficients for the Legendre polynomials corresponding to  $C^{13}(d, \alpha)B^{11}$  angular distributions.

for the polynomial expansion are also shown in the figure. The values of the coefficients, together with the error estimate  $(\chi^2/d)^{1/2}$ , are tabulated in Table I. This parameter is a measure of the goodness of fit and should be unity for a "perfect" agreement.<sup>7</sup> The deviations from unity indicate that either the assumed functional dependence is inadequate or that the assigned errors are too conservative; in particular, that they have been underestimated by the factor  $(\chi^2/d)^{1/2}$ . In performing the least-squares fit, the errors assigned were standard statistical deviations and are, therefore, somewhat low. The statistical uncertainties were, in general, about  $3\%$ . Errors could also arise from nonuniform target thickness, from lack of precision in setting the angle, and from the small number of background counts present in the spectrum. It is estimated that these could contribute an additional  $1\%$  error in the results. Thus, the errors assigned to the data points should be increased by a factor of about 1.25. This decreases  $(\chi^2/d)^{1/2}$  by the same factor, which leads to reasonable fits and, hence, to good repeat probabilities. On this basis it is concluded that the assumed functional dependence is consistent



FIG. 4. Variation with energy of coefficients obtained in leastsquares fits to  $C^{18}(d, \alpha)B^{11}$  angular distributions. Each box shows the variation with energy of one of the coefficients in the Legendre polynomial expansion.

with the data and that the indication of a poor fit is simply a result of underestimation of experimental error.

The angular distribution of the outgoing wave from a nuclear reaction can be expressed in terms of a polynomial in  $\cos\theta$  of order equal to or less than 2l, where l is the maximum orbital angular momentum of the incident wave which contributes to the reaction.<sup>8</sup> Thus, the presence of a significant  $P_6$  term in the polynomial expansion indicates that there must be appreciable nuclear penetration by the *1=3* partial wave of the incident particle. To check this, computations of penetration factors were made using the relationships given by Blatt and Weisskopf.<sup>9</sup> The values used for the radial functions were those of Bloch *et al.*<sup>10</sup> Use of a nuclear radius of  $3 \times 10^{-13}$ cm gave a penetration of about  $\frac{1}{2}\%$  for the *l* = 3 partial wave at a deuteron energy of 3.2 MeV. This implies that the level, or levels in the compound nucleus must be strongly excited.

Figure 4 shows the variation with energy of each of the coefficients obtained in the least-squares fitting. These numbers completely parametrize a large number of data in terms of a compound-nucleus reaction. Therefore, such a plot provides a convenient way to search for the presence of resonances in the compound nucleus which are not sharp enough, or strong enough, to be identified in the excitation functions. When analyzed this way, an extensive series of measurements of angular distributions might yield information, not otherwise accessible, on the spin and parity of the states involved in the compound nucleus, since the amplitude and phase of contributing Legendre polynomials will vary with energy.

# $B^{11}(He^3, p)C^{13}$

The angular distributions for the B<sup>11</sup>(He<sup>3</sup>,  $p$ )C<sup>13</sup> reaction at laboratory energies of 8.6, 9.6, and 10.3 MeV are shown in Fig. 5. The differential excitation functions at laboratory angles of 35° and 140° over the energy range of 8 to 11 MeV are shown in Fig. 6. The error bars indicate statistical uncertainty only. The absolute value of the cross section is accurate to within  $15\%$ , the un-

<sup>8</sup> C. N. Yang, Phys. Rev. 74, 764 (1948).<br><sup>9</sup> J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).<br><sup>10</sup> I. Bloch, M. H. Hull, Jr., A. A. Broyles, W. G. Bouricius,<br>B. E.

<sup>£.</sup> <sup>7</sup> P. Cziffra and M. J. Moravcsik, Lawrence Radiation Laboratory Report, UCRL-8523 Rev., 1959 (unpublished).

certainty residing mainly in the assessment of the target thickness.

The structure in the forward peaking of the angular distributions, and the absence of sharp resonances in the yield curves, suggest that the reaction proceeds mainly by a direct process. Because of this, the data have been analyzed on the basis of a two-particle stripping reaction. It was noted at the outset that the rise of cross section at backward angles might be indicative of a compound-nucleus contribution, of distortion, or of backward stripping, but all these effects were disregarded in the following analysis.



FIG. 5. Angular distributions of protons from the  $B^{11}(He^3, p)C^{13}$ (ground state) reaction at bombarding energies of 8.6, 9.6, and 10.3 MeV. For clarity the statistical error bars are shown on only a few representative data points. The dashed curves are fits to the forward portions of the data obtained from Eq. (2) of the text. The solid curves are results of numerical integration of Eqs. (3) and (4) of the text.

The theoretical aspects of two-particle stripping have received considerable attention recently. Tobocman<sup>11</sup> and Butler<sup>12</sup> obtained expressions for the angular distribution, which have the form  $g(\theta)$   $[j_L(kr_0)]^2$ , by treating the incident particle as two lumps, one of which is captured by the target nucleus. (The notation used here is conventional. In particular it follows that of references 13 and 14.) Such a treatment should be valid if the finalstate nucleus has the parentage of the target nucleus



FIG. 6. Differential yield curves for the B<sup>11</sup>(*d*,  $\alpha$ )C<sup>13</sup> (ground state) reaction as recorded at laboratory angles of 35° and 140° over the energy range from 8 to 11 MeV.

plus the captured lump; i. e., if the final nucleus is well described by the cluster model.

It is clear that a more general treatment should include the structure of the projectile and the stripped lump. Various attempts have been made to do this.13-16 These investigations yield results which are similar to those obtained in the simpler stripping case; namely, the angular distribution appears in the form

$$
G(\theta)\sum_{L}|A'(JLSl_1l_2)j_L(kr_0)|^2.
$$
 (1)

The structure of the captured particles shows in that  $l_1$  and  $l_2$ , the *l* values for the captured particles, are explicitely contained in the expression for the cross section. It is assumed that the captured particles are in states of definite orbital momentum. This assumption is not essential to the argument, but it simplifies the resulting expressions. Since the coefficient *A'* involves only recoupling coefficients and coefficients of fractional parentage, analysis of two-particle stripping reactions can potentially yield information on the shell-model structure of nuclei involved in the reaction.

If the common approximation is made that the capture occurs entirely at the nuclear surface of radius  $r_0$ , and a Gaussian is taken for the He<sup>3</sup> wave function, the following expression is obtained  $17$  for the case investigated here, in which *L* can take but two values, 0 or 2,

$$
\sigma(\theta) = \exp(-K^2/4\gamma^2) \{a_0[j_0(kr_0)]^2 + a_2[j_2(kr_0)]^2\}.
$$
 (2)

<sup>13</sup> M. El Nadi, Proc. Phys. Soc. (London) **70**, 62 (1957).<br><sup>14</sup> H. C. Newns, Proc. Phys. Soc. (London) **76,** 489 (1960).<br><sup>15</sup> N. K. Glendenning, Lawrence Radiation Laboratory Report UCRL-9505, 1960 (unpublished).

<sup>16</sup> C. F. Clement (private communication).<br><sup>17</sup> B. B. Marsh, Ph.D. thesis, University of Rochester, 1962 (unpublished), pp. 47-51.

<sup>11</sup>W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961).

<sup>12</sup> S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957).

If one does not make the assumption that the capture occurs at a sharp value of *R,* the radial part of the integral cannot be separated as is done to obtain Eq. (2). It then becomes necessary to make other assumptions concerning  $V_n$  and  $V_p$  before proceeding with the integration. Such an approach has been devised by Macfarlane and French.<sup>18</sup> Their expression for the differential cross section contains matrix elements which must be evaluated numerically. For the specific  $B^{11}(He^3, p)C^{13}$ case at hand and under the assumption that both nucleons are captured with  $l_1 = l_2 = 1$ , again using a Gaussian wave function characterized by the size parameter  $\gamma$ , following expressions are obtained<sup>19</sup> for the two,  $L=0$ and *L=2,* matrix elements:

$$
(H_{110})^{1/2} = \frac{D^2}{2a\pi^{3/2}3^{1/4}24^{1/2}\gamma^3 r_0^5} (I_0 - I_1)
$$
  
\n
$$
\times \exp\left(-\frac{\kappa^2}{8\gamma^2 r_0^2} + \frac{a^2}{24\gamma^2 r_0^2}\right),
$$
 (3)

and

$$
(H_{112})^{1/2} = \frac{D^2}{a\pi^{3/2}3^{1/4}48^{1/2}\gamma^3 r_0^5} (I_1 - I_2)
$$

where

$$
I_{0} = \int_{0}^{\infty} dr \, j_{1}(r) r^{2} \exp\left(-\frac{r^{2}}{12\gamma^{2}r_{0}^{2}}\right)
$$
\n
$$
\times \int_{1a-r}^{a+r} dR \, j_{1}(R) (R^{2} + r^{2} + \eta^{2}) P_{0}\left(\frac{r^{2} + a^{2} - R^{2}}{2ar}\right)
$$
\n
$$
\times \exp\left(-\frac{R^{2}}{12\gamma^{2}r_{0}^{2}}\right), \quad (5) \quad L = \frac{1}{2a}
$$
\n
$$
I_{1} = a \int_{0}^{\infty} dr \, j_{1}(r) r \exp\left(-\frac{r^{2}}{12\gamma^{2}r_{0}^{2}}\right)
$$
\n
$$
\int_{1a-r}^{a+r} dR \, j_{1}(R) (R^{2} + r^{2} + \eta^{2}) P_{1}\left(\frac{r^{2} + a^{2} - R^{2}}{2ar}\right)
$$
\n
$$
\times \exp\left(-\frac{R^{2}}{12\gamma^{2}r_{0}^{2}}\right), \quad (6) \quad \text{pa}
$$
\n
$$
I_{2} = \int_{0}^{\infty} dr \, j_{1}(r) r^{2} \exp\left(-\frac{r^{2}}{12\gamma^{2}r_{0}^{2}}\right)
$$
\n
$$
\times \int_{1a-r}^{a+r} dR \, j_{1}(R) (R^{2} + r^{2} + \eta^{2}) P_{2}\left(\frac{r^{2} + a^{2} - R^{2}}{2ar}\right)
$$
\n
$$
\times \exp\left(-\frac{R^{2}}{2ar}\right), \quad (7) \quad \text{th}
$$

18 M. H. Macfarlane and J. B. French (private communication). 19 For an outline of the Macfarlane-French approach to twoparticle stripping and for a detailed derivation of expressions 3 through 7 of this text, see B. B. Marsh, Ph.D. thesis, University of Rochester, 1962 (unpublished), pp. 58-79.

 $12\gamma^2 r_0^2$ /

with  $a = qr_0$ ,  $\eta = (2\mu B)^{1/2}r_0$  ( $\mu$ =nucleon reduced mass, *B—*binding energy of the nucleon in the final state with respect to the initial state),  $\kappa = Kr_0$ .<sup>19</sup>

The integrals in Eqs.  $(5)-(7)$  are familiar in solid state and molecular physics where they are known as twocenter integrals.<sup>20</sup> To evaluate the integrals, a program has been written for an IBM 7070 computer using Simpson's rule for numerical integration. The results of the integration for the B<sup>11</sup>(He<sup>3</sup>,  $p$ )C<sup>13</sup> case are plotted as solid lines in Fig. 5, along with the dashed curves obtained from application of Eq. (2). It is seen that both sets of curves provide reasonable fits to the forward portions of the angular distributions.

It is somewhat surprising that the results of the two basically different approaches show such close similarity. This similarity is exhibited in detail in Fig. 7, where the factors determining the shape of the curves in the two cases are plotted side by side for two different values of the radial parameter. The resemblance of the two sets of curves can only mean that in spite of a different physical approach the final expressions are related analytically. This point is being investigated further.

It must be emphasized that the theoretical treatments referred to here are all based on plane-wave Born approximation and do not take into account distortion effects. This is a serious shortcoming, since such effects seem to be more pronounced in  $(He^3, p)$  and  $(\alpha, d)$ reactions than in  $(\tilde{d}, p)$  reactions. The main justification for the simplification is that it provides some information on the salient features of the reactions without requiring extensive high-speed computer work. However, the significance of fits obtained with such planewave approaches must be viewed with discrimination.

In Fig. 5, the relative contribution of the *L=0* and *L*=2 terms is indicated by the ratio  $a_2/a_0$ . Clement<sup>16</sup> has calculated the expected value of this ratio for different coupling schemes. The ratio has a maximum value of 2.0 for *j-j* coupling, and decreases as one goes to *L-S*  coupling. The experimental ratio, about 25, is inconsistent with the theoretical prediction. This discrepancy is most likely due to the oversimplified nature of the reaction theory used in which distortion effects are neglected. However, the disagreement may be due, at least partly, to the uncertainty in the value of the size parameter  $\gamma$ . The curves in Figs. 5 and 7 are calculated using  $\gamma^{-1} = 6$  F. The proper value of  $\gamma$  is not well known. Since  $\epsilon$  the assumed Gaussian He<sup>3</sup> function is only approximate there is no *a priori* reason for assuming that  $\gamma$  should have the same value for both the  $L=0$  and  $L=2$  contributions. As shown in Fig. 8, where  $H_{110}$  and  $H_{112}$  are plotted for various combinations of  $r_0$  and  $\gamma$  parameters, the shape of the angular distribution is not very sensitive to the value of  $\gamma$ , but the magnitude is. Thus, one can get large differences in the ratio with but little effect on the angular distributions by choosing different values of  $\gamma$ for the two contributions.

<sup>&#</sup>x27; P. P. Lowdin, Advan. Phys. 5, 1 (1956).





#### **CONCLUSIONS**

It is interesting to note the similarity in the results of the treatments of the two-particle transfer reactions by the Newns and the Macfarlane-French approaches. Only minor differences arose in the fitting of the  $B^{11}(He^3, p)C^{13}$  experimental angular distributions by Bessel functions and by the numerically integrated curves. It is concluded that the additional approximations made by Newns within the framework of the planewave Born approach are justifiable.

Another point is the appearance of terms as high as sixth order in the Legendre polynomial fits to the  $C^{13}(d,$  $\alpha$ )B<sup>11</sup> data. Under the assumption that this reaction proceeds mainly via formation of  $N^{15}$ , this implies that partial waves as high as  $l = 3$  contribute significantly to the reaction, despite the fact that computations of penetrability factors indicate that less than  $1\%$  of the  $l=3$ partial wave penetrates the nucleus. The inference is that the states in the compound nucleus excited by this partial wave participate strongly in the reaction process.

Furthermore, it is concluded that the overlap of momentum transfer regions for the reactions related by the transfer of similar clusters does not necessarily imply similar reaction mechanisms, at least at energies available for this work. It would be interesting to see if higher bombarding energies would yield results permitting a comparison of configurations involved in reactions of the type  $X(\text{He}^3, p)$ *Y* as against  $X(\alpha, d)$ *Y* or  $Y(d, \alpha)X$ . In the absence of compound-nucleus contributions, the main differences would be expected to stem from the effects of spin selection rules, which permit both triplet and singlet deuteron transfer in the case of  $(He^3, \rho)$  and only triplet in the case of the  $(d, \alpha)$  reaction. Notwith-

FIG. 8. Matrix elements  $H_{110}$  and  $H_{112}$  plotted for various combinations of parameters  $\gamma$  and  $r_0$ . Although the shape of the curves is not very sensitive to the value of  $\gamma$ , the relative size is. By choosing different values of  $\gamma$  for<br>the  $L=0$  and  $L=2$  contributions, a large difference in the ratio of the contributions can be obtained while retaining a reasonable fit to the forward portions of the data of Fig. 5.



standing this, it appears that distorted wave calculations will be necessary to arrive at firm conclusions.

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# Symmetry of Neutron-Induced U<sup>235</sup> Fission at Individual Resonances. II

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Neutrons from the Project Gnome nuclear test were resolved in energy by time-of-flight and used to study symmetry of fission in U<sup>235</sup> in the resonance region. Symmetry was deduced by measurement of the fission product ratio Ag<sup>111</sup>/Mo<sup>99</sup> at resonances from 8.8 to 40 eV. The maximum decrease observed in the probability of symmetric fission as compared to the probability in thermal fission of  $U^{235}$  was  $50\%$ . The maximum increase observed was 22%. Four levels showed an increase in symmetry. Thirteen levels showed a decrease in symmetry. With the exception of one level at 15.4 eV, assignments of levels by symmetry are in agreement, in the region covered, with previous work.

### **INTRODUCTION**

THE Project Gnome explosion of a nuclear device<br>on 10 December 1961 has been used for further<br>development of the neutron time-of-flight technique HE Project Gnome explosion of a nuclear device on 10 December 1961 has been used for further described in a previous paper.<sup>1</sup> An experiment similar to other so-called ' 'neutron wheel" experiments sponsored by this laboratory was performed in order to ascertain the feasibility of recovery of target material from the vicinity of an underground explosion and, by improvement of over-all neutron energy resolution, to obtain more precise data on the symmetry of fission of U<sup>235</sup> at individual resonances. Successful recovery of the exposed U<sup>235</sup> in an undamaged condition was regarded as a necessary assurance for safety in the planned use of Pu<sup>239</sup> in a follow-on experiment. In the case of Pu<sup>239</sup>, large scale damage to the target would create an unacceptable level of alpha contamination in the immediate environment.

It was expected that an underground explosion, as compared with an atmospheric explosion, would have the following features bearing on the application of neutron time-of-flight techniques to nuclear research:

1. Since the device is almost totally shielded, scattered neutrons contributing to the background at the target are minimized.

2. Fallout is avoided.

3. Under the most favorable conditions, recovery of the target material is simplified. On the other hand, containment can lead to venting down the neutron pipe with possible destruction of the target. If the destructive effects are minimized by containment, then it becomes possible to maximize the device yield and, consequently, the neutron flux over a given flight path.

This experiment was designed to improve on the resolution of the previous experiment by a factor of three without significant changes in counting statistics. Previous information on symmetry of fission at individual resonances indicated a possible correlation of symmetry with spin. It was assumed that, if  $ideas^{2-4}$ relating spin to fission symmetry are correct, a sufficient improvement in the resolution of individual levels in the epithermal region would demonstrate that strong resonances are characterized by one or another of two possible fission product yield distributions. If more than two possible distributions exist, the explanation that the effect is due only to spin changes at the resonances might be excluded. The previous wheel experiment demonstrated that symmetry of fission varied from resonance to resonance but did not demonstrate the existence of only two possible distributions.

<sup>\*</sup> Work performed under the auspices of the U. S. Atomic Energy Commission. 1 G. A. Cowan, Anthony Turkevich, C. I. Browne, and Los

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